

2.4 Remainder Theorem & Factor Theorem

Remainder Theorem: If a polynomial is divided by $x-a$, then the remainder is $P(a)$.

Ex 1: Determine the remainder for

$$(4x^2 - 2x + 7) \div (x-1)$$

$$\begin{array}{r} x-1=0 \\ +1 \quad +1 \\ \hline \end{array}$$

$$4x^2 - 2x + 7$$
$$4(1)^2 - 2(1) + 7 = \boxed{9} \leftarrow \text{remainder}$$

$$x = \boxed{1}$$

Ex 2: Determine the remainder for

$$(m^8 + 6m^5 - 5) \div (m+2)$$

$$\begin{array}{r} m+2=0 \\ -2 \quad -2 \\ \hline \end{array}$$

$$m^8 + 6m^5 - 5$$
$$(-2)^8 + 6(-2)^5 - 5 = \boxed{59} \leftarrow \text{remainder}$$

$$m = \boxed{-2}$$

Factor Theorem: If $P(a) = 0$, then $x-a$ is a factor of the polynomial.

Ex 3: Is $x-5$ a factor of $2x^3 - 18x^2 - 2x + 210$?

$$\begin{array}{r} x-5=0 \\ +5 \quad +5 \\ \hline \end{array}$$

$$2x^3 - 18x^2 - 2x + 210$$
$$2(5)^3 - 18(5)^2 - 2(5) + 210 = \boxed{0}$$

$$x = \boxed{5}$$

yes, $x-5$ is a factor!

Ex 4: Is $x+8$ a factor of $9x^3 + x - 7$?

$$\begin{array}{r} x+8=0 \\ -8 \quad -8 \\ \hline \end{array}$$

$$9x^3 + x - 7$$
$$9(-8)^3 + (-8) - 7 = \boxed{-4623}$$

$$x = \boxed{-8}$$

no, $x+8$ is not a factor.